**Fluid Mechanics**
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Lecture – 17
Fluid Kinematics

Welcome you to this fluid mechanics class on to fluid kinematics, this is second lectures on fluid kinematics, this is what quite interesting lectures what today I will cover it with starting from the derivations of vorticity to; so the real time the vortex formations in Bay of Bengal which is super cyclones what is happening today. So, let us have a these lectures which started from very basics how the vorticity is there and how the fluid rotations we are talking about that.

And where very larger scales is meso scale process of cyclones which can looks like a vortex process, a very real time near real time vortex formations what is available in Internet what I am going to show you, so this is quite interesting lectures for you to locate the vortex from smaller scale to the bigger scale.

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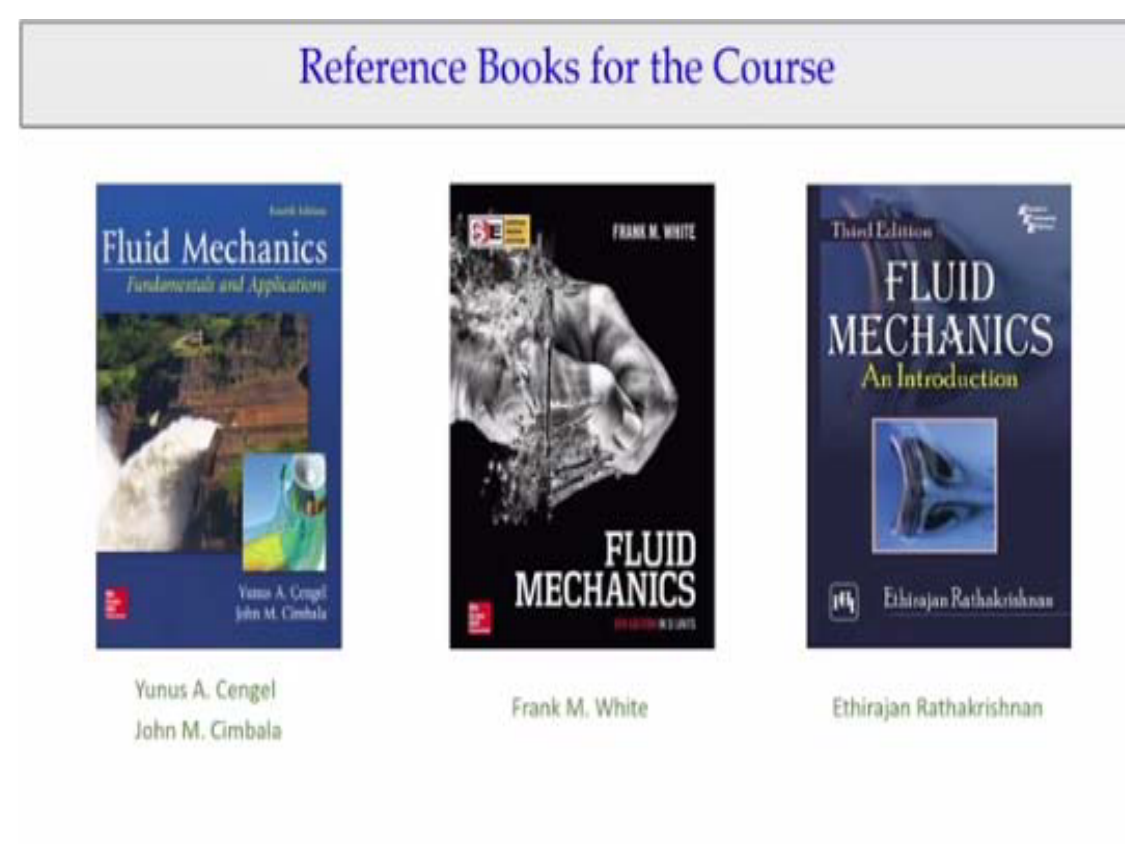
Today, I will cover again these 3 books and mostly I will talk about recently, we are talking about more Rathakrishnan's book and Cengel, Cimbala, so it is a good book on fluid kinematics.

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7. Summary

Now, let us have the contents of the today lectures, I will very interestingly, I will show the experimental facilities what is there in Department of Chemical Engineering, IIT, Guwahati, particle imagine velocity meter which measures 3 dimensional velocity components, so when you measure the 3 dimensional velocity component, then you can understand how vortex formations happens, how the turbulence characteristics happens.

So, which is more important today's world to know it very, very micro scales the process to understanding as compared to macro scales or cross characteristic things what we did it in 20 years back or 30 years back, then again another examples from the; as a CFD solvers, the collapse of water columns with a obstacles that was very interesting will be surface flow that is what we will see it.

Then, we will go for, already I discuss about the Lagrangians or Euler descriptions, I will just touch upon that so, in these the flow patterns what we are getting it then, we will go for how the motion and deformation of the fluid elements, vorticity and rotational and some few examples best for the motion and deformations and followed by the summary.

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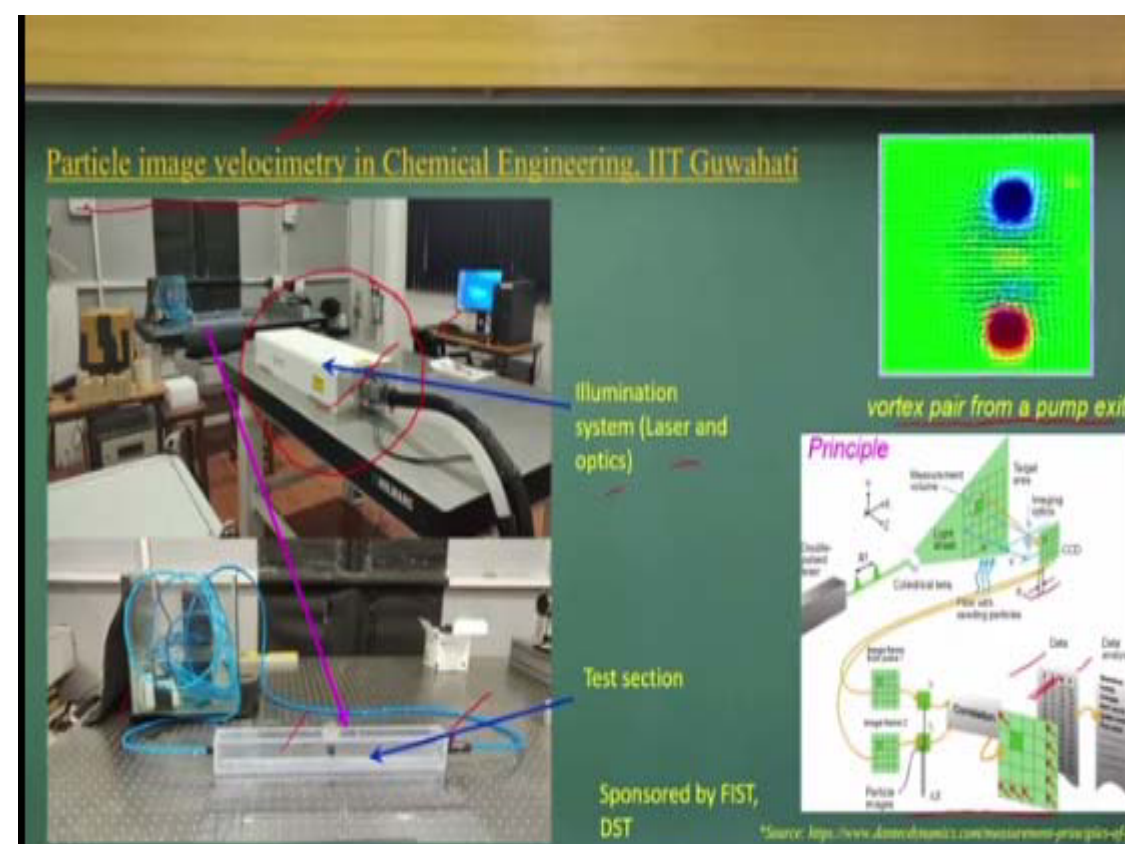
Now, if you look at the facilities what you have granted from the Department of Science and Technology come out of India, this facility is known as the particle image velocity materials, so where this is the facility we generate the laser beam, okay so and that laser beam passed through the test sections which has a 2 cameras to monitors how these the laser beams are changing it with an image processing, we can compute the 3 dimensional velocity fields.

I am not going more detail how we what is the basic principle of image; particle image velocity materials but there is instrument like a laser beams, then the test sections and the 2 cameras based on that with these principles, what we have given here to monitor the particles at different time frames, we can obtain the 3 dimensional the velocity fields, as you see it is very interesting photographs what we are coming from experimental data.

Because here you can get 3 dimensional velocity fields, if you look into that there are 2 vortex are forming just from a pump exit and the you can see these factors, you can see the vortex sheddings which is going down and 2 pair of vortex formations are there and so these quite interesting figures of vortex formations and the propagations of vortex pair that what we can go it in a very detailed way, if you have a facility like PIV or particle image velocimetry.

So, these very unique facilities that is what we have in a Department of Chemical Engineering IIT, Guwahati, so but what I am to tell it at present, we have the facilities to measure the 3 dimensional the velocity fields with very accurately, so that we can represent like this type of vortex forces what it happens it that can be possibility.

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Applications

Collapse of Water Column with Obstacle
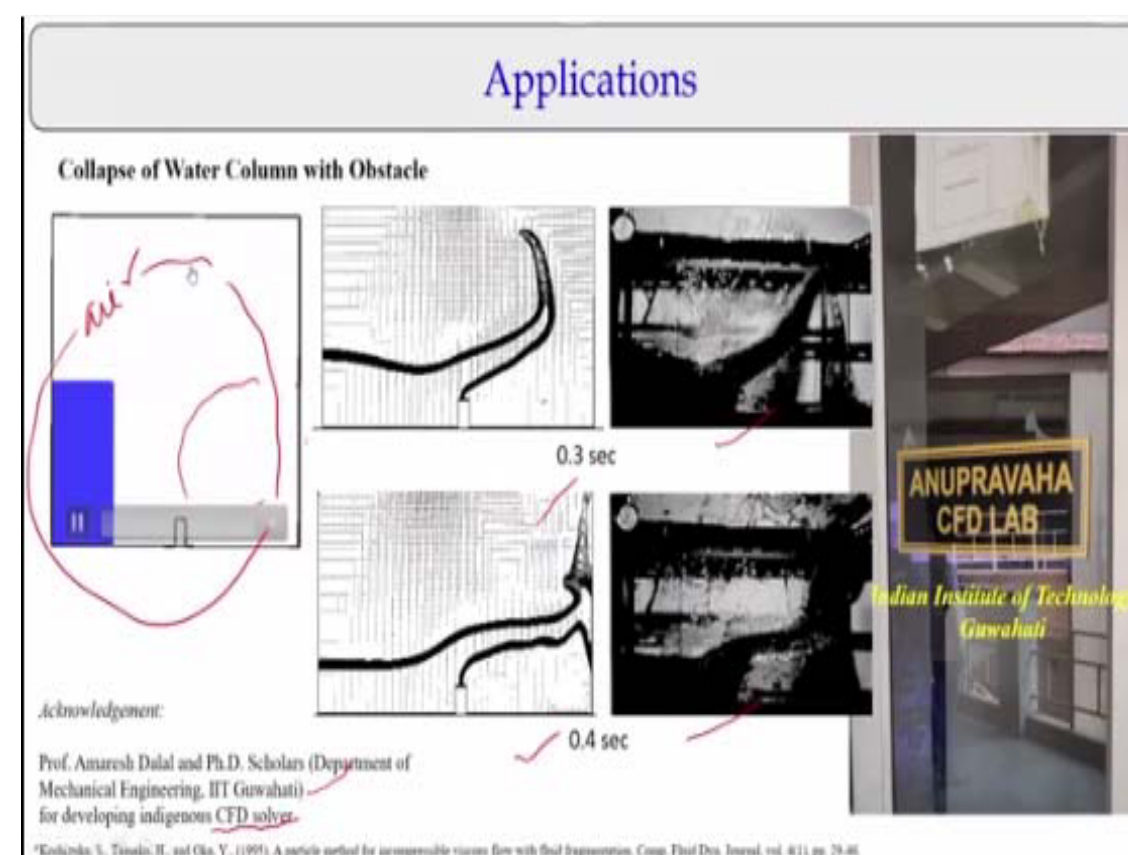
Now, let us go to the one of very interesting again from the acknowledging see professor Dalal and students group in Department of Chemical Engineering, IIT, Guwahati who has developed indigenous CFD solvers, now if you look at this free surface, you can see that how interesting things is are happening here, you just see these collapse of water column with the obstacles.

That means, you have water columns that maybe have a water tanks is collapsing it, then there is obstacles, how the flow patterns happens, if you look at these free surface is very interesting things what is happening it and this the CFD solver today's it is able to capture these so complex process if you look at how the complex processes are happening it, how the mixing of water and air is happening it.

So, if you look it at these are things, it is a really possible today's now it is not that difficult and this is what is showing with a comparison with the velocity fields and all with the experimental data sets. So, if you look at this way, there are very complex processes also we can get the streamline patterns, velocity pattern and the pressure patterns (()) (06:59) surface variations patterns either using CFD solvers or having an experimental facility with particle image velocimetry.

So, if you have that you can get the solutions that so complex solutions also, we can get it and you can see how the pressure variability is there, how the velocity variability is there, how the density varies there and these are the problems where there is a mixing of the 2 domain problems, this is the water is there also, air is there so, how these 2 mixings are happening it

and that is what the complexity of the problems and which is (()) (07:44) say possible to do this type of the studies and which will give a very interesting thing.

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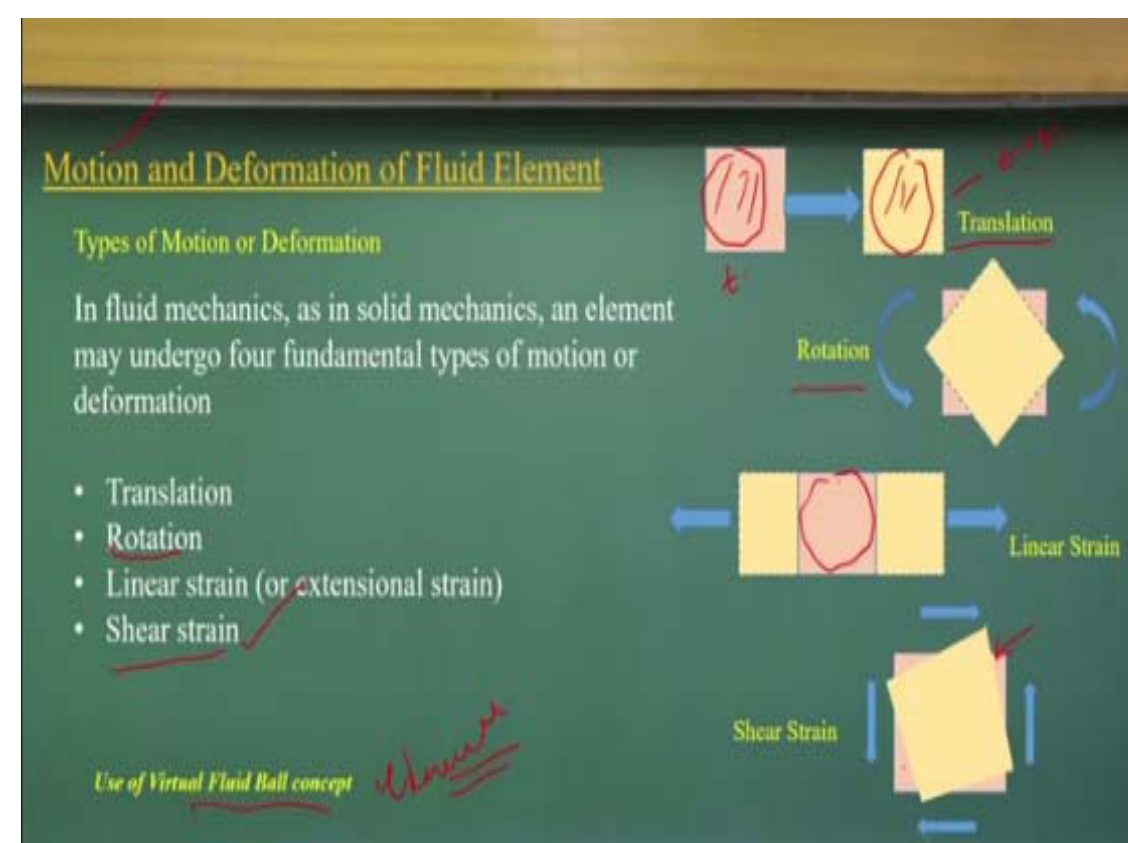
Let us come back to our basic fluid mechanics okay, so as in the last class I talked about 2 types of descriptions; one is Eulerian frame of descriptions, another is Lagrangian frame of description and that the descriptions we try to visualize as form of virtual fluid balls okay, so today I will talk about that I have the fluid element which is you know in any of fluid mechanics book.

They talk about the fluid element which is representing a certain space of the fluid particles which is much larger scale than the molecules levels or it is not that bigger scale to represent the flow process, so mostly I will talk about which is calling virtual fluid balls, okay. So, when you have a balls then it can have go through any type of motions and the differences like it can be translations like variations that you have a ball, it can be translated, it can go in a u direction or v direction.

Or it can have a displacement in a x direction, y direction, z direction and if the displacement is caused because of the velocity component, so that means if this the virtual ball, it is at the t time and this is a t plus delta t time, we will have more detail discussion how much it will travel it that depends upon the velocity at the time t and at after Δt, what could be the displacement which is simple things, the displacement is equal to velocity into Δt.

So, it is very easy concept that how much of displacement will be there, how much of translations will be there and the particle; the virtual fluid balls are the element; the fluid elements can go for a rotation that means, there is no translations motions that means, it is just go for a rotation, it is there, only it is goes for either clockwise it represents or the anti-clockwise it represents.

So, the fluid element can go for a rotation, so that means we can compute what could be this angular velocity, if it is going through a rotation; the clockwise rotations or anti clockwise rotations. So, if you try to understand that these rotations all depends upon the velocity fields, the velocity field the variations will cause the rotations of these fluid particles.

The velocity of the field also talk about how the translations motion will happen from point A to point B, as fluid particles are changing at the time t to t plus Δt, so all it depends upon the velocity fields, so that way we can have a translations or the rotations which is motions part, then we talk about this fluid element as in the solid, it could have the deformations, like it can have a linear deformation, like it can be elongated.

So, that means if you have given a just stress, okay or you compress it, so that means the elemental sides; the volume of the fluid element that may increase it or can decrease it that depending upon also the velocity distributions, how we have that is what will be trace the fluid elements or the compress the fluid element or if I have a virtual ball so, the basically the dimensions of balls will change.

See, if I have a virtual ball concept that means, what I am talking about that the dimensions of the ball can change it okay, if I have a give a linear strain okay, positive strain it can expand it, if I give the compressing it, then it will be reduced the size, the similar way we can have a shear strain, we can have a the shear strain to the fluid particles like, you can have a shear strains to the particle.

So, there is a difference between this rotation and the shear strains okay, you can see the graphical how they are different okay, we will discuss more. So, basically if you look at that the fluid can go through 2 type of motions; translations and rotations also, it can have a deformations of linear strains and the shear strain.

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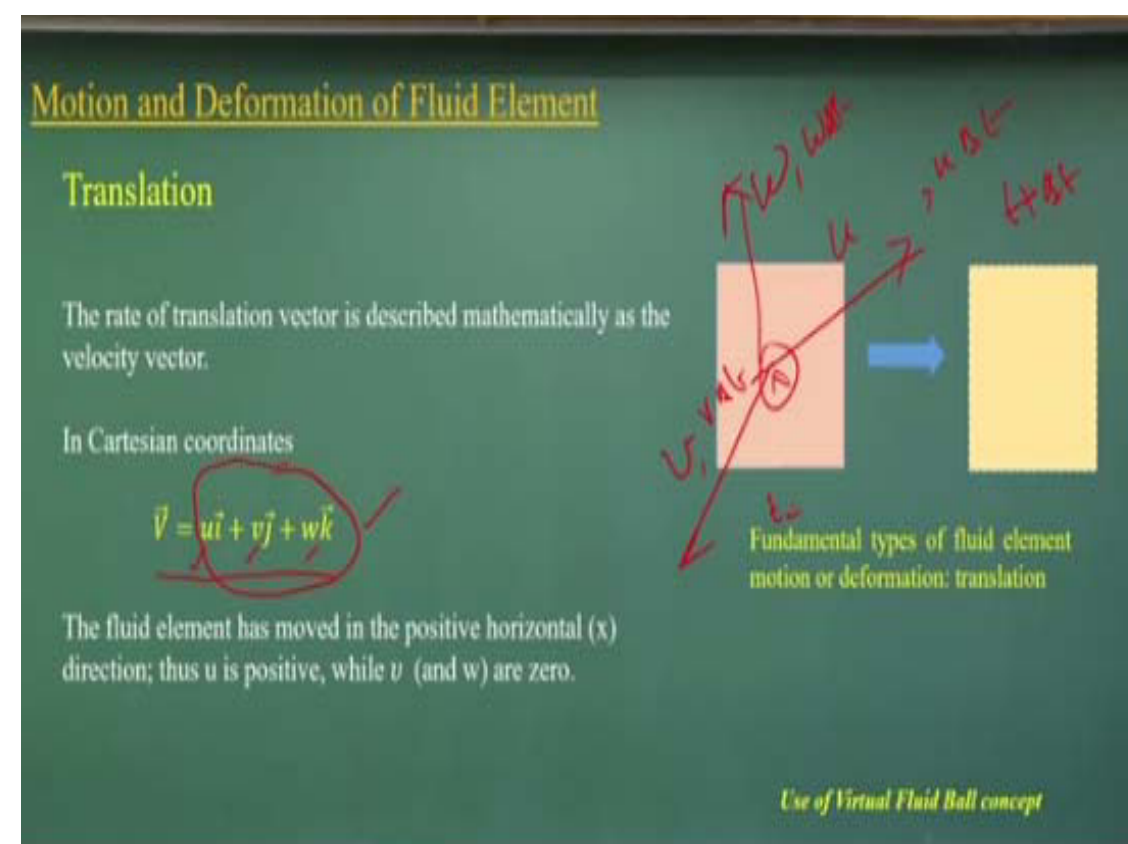
Now, come to the translations which is very easy concept okay, which is the velocity factor which is responsible for shifting a fluid particle from A location to B locations, it depends upon the velocity components like in this case, you have a small u, v and w is a scalar velocity component in x, y, z directions respectively.

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

So, that means at the Δt time, after the Δt times, these particles which is there it can move it at a displacement of u into Δt, v into Δt, w into Δt.

So, that way it will move in these 3 directions to get a new positions factors, so it is a simply translations from A point to the B point with y the velocity vector components, okay that means this particles has the velocity vector component u, v, w and at the t time, t plus delta t time, it will move it with a in the x direction would be uΔt and vΔt and ωΔt from these origins locations.

So, it is a very simple thing as you know it in solid mechanics okay, if a particles are moving with a u, v, w what could be the display positions vectors at $t + \Delta t$ times, so it is very easy we can have this ones, okay but many of the times we see the 1 dimensional components, other components you may consider it is a 0.

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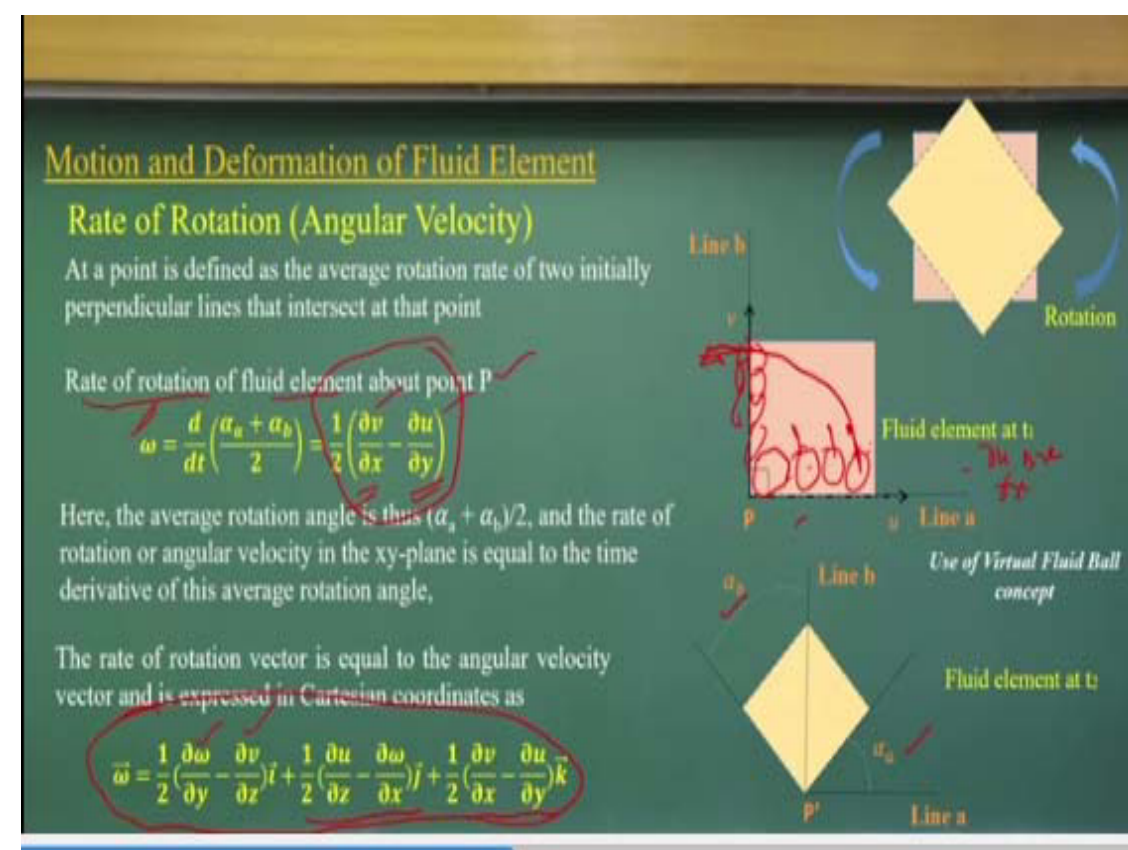
Now, coming to the rate of rotations, which is called angular velocity okay, what could be the angular velocity of the fluid point. See if I take it this is the fluid element okay, it will have a this the velocity variations that means, if this is the u velocity is here at this point, you will have $u + \frac{\partial u}{\partial x}\Delta x$, as the x direction is changes. If this is the v velocity here at this point we will have a $v + \frac{\partial v}{\partial y}\Delta y$.

What I am telling it if I have a u and v, it is a 2 dimensional element where we have consider it is easy to understand it and it is the z direction which is a perpendicular to this surface, so that way it will have a rotations along the z directions; along the z directions perpendicular to z directions, it will have a rotation in on xy plane. So, if is I have a, u and v velocity at this point, I will have a velocity at this point will be $u + \frac{\partial u}{\partial x}\Delta x/2$.

So, similar way at this point I will have the velocity, the v; $\frac{\partial v}{\partial y}\Delta y$ but this will be the u velocity, so similar way I can write the what could be the velocity at this point. So, now if you look it when you take a fluid element in the 4 corner points, we have the velocity variation because of this velocity component okay, because of this velocity component and this velocity component, the fluid particles is going to rotate it, is it correct?

The velocity component of the v at these locations that what will we try to rotate it, the velocity component v, u the gradient the relative difference of velocity component will be move it, so that way what we can have that how we can compute the relative velocity difference between

these point that what multiplying with the dt will give a distance and then we can get it what will be the angular rotations, very simple way.

$$\omega = \frac{d}{dt}\left(\frac{\alpha_a + \alpha_b}{2}\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

If you want to have very detailed derivations writing from the Taylor series, find out in the distance travels that computing's all these component of the velocity at each points, you can follow of any of my wave lectures which is there in NPTEL's, so but let us not I spend more time on that, let me take you very concept wise that the angular rotations, the rate of rotation of the fluid element about the point P will be the $(\alpha_a + \alpha_b)/2$ which we can easily write in terms of the partial derivative of B and the u component, the scalar component velocity.

It can be easily written it, if you follow the Taylor series and with these things okay, this is what that means this is the angle of rotations is given in a functions of in terms of the scalar; the partial derivatives of the scalar component of P and u in a x and y directions respectively but if you want to make it 3 dimensional like if you look at this fluid particles okay, I just computed for the z directions okay.

Similar way, I can find out what could be the rotations in the x directions and the y directions; that what will come it the similar like this, so this is what the rate of rotations factors, it has x component rotations, the y component rotations and z component, these are complete rotations factors is in terms of the change of the velocity gradient in x, y, z directions.

So that; so please try to have a simple relationship to try to know it whenever you are getting the compute the rotations in the y; z plane you need to have a scalar gradient component in x and y.

$$\vec{\omega} = \frac{1}{2}\left(\frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

Similar way, if you want to have these, computing these rotations in the ith plane that means you are looking at the scalar gradient A of z and the y plane, so those things you can just re-remember it or it is not necessarily you try to simplify it as get a fluid elements and you just think it.

But if you come to the virtual ball concept, the same concept I can tell it there will be virtual ball concept. What I am talking about now that not the fluid elements, it is a virtual fluid balls that means, there will be a series of balls will be there, since there is a velocity gradients are there, so these balls will try to move these directions, these balls have try to move to these directions.

Because of that there will be start a rotation, the direction can change it, they can have a clockwise direction so basically, if you think it there are the balls are there as they have the different the velocity gradients are there, so they will move different displacement, as a jointly, we can see there is a rotation, what is happening it, as a rotation what is happening in a z plane as well as we can compute for x and y directions.

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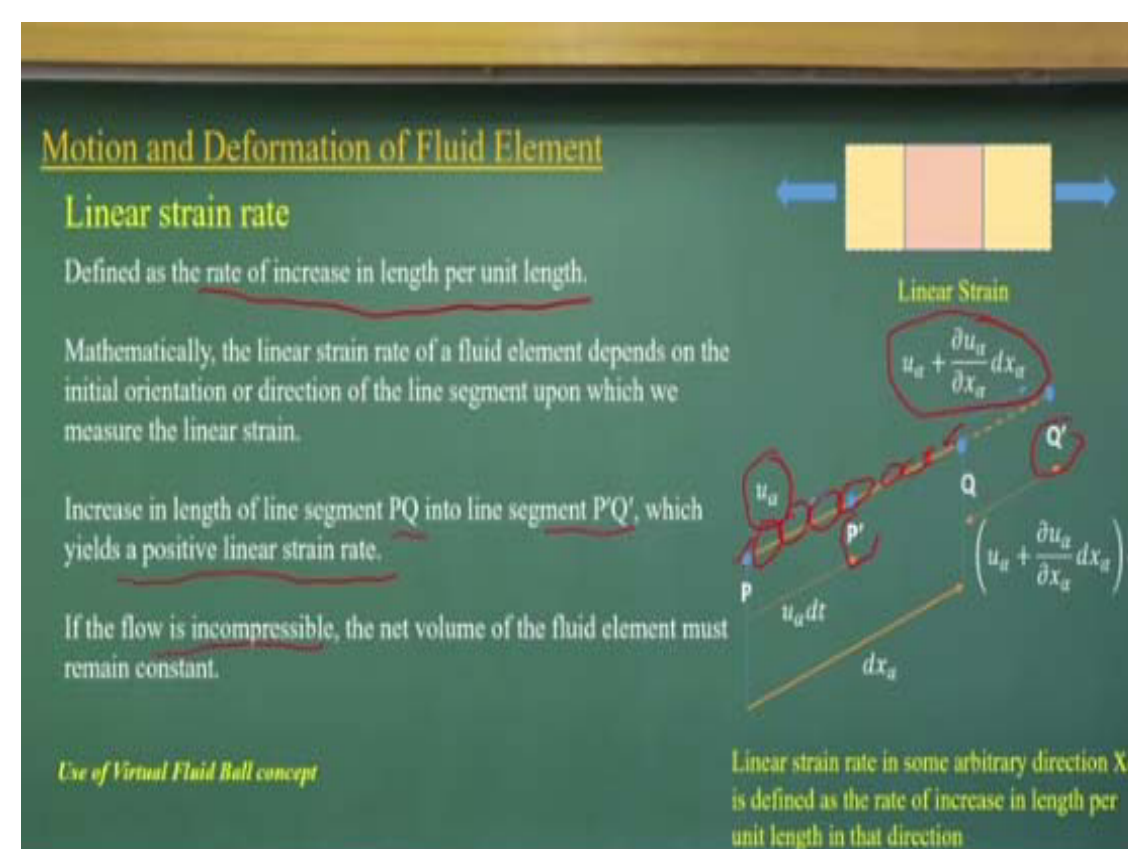
Now, coming to the very simple thing is called linear strain rate that means, what is the strain rate; rate of increase the length per unit length, this is very simple definition, okay. So, rate of increase in length for unit length that means, let you have a the initially, you have a the fluid element which is starting from P to Q okay, at the point P you have the velocity u_α but at the Q, we should have a based on the Taylor series, you can have this velocity.

$$u_\alpha + \frac{\partial u_\alpha}{\partial x_\alpha} dx_\alpha$$

At the dt times, these P point will be moved to P′, the Q point will move to Q′, the distance between these 2 will the different, so that is the reasons they will be linear strain, distance travelled by this P and Q will be the different as the velocities are the different at the P and Q,

if the velocity difference between the P Q not there, then you will may not have a linear strain, will not have a linear strain.

Since, there is a velocity variation, the fluid element which connected as a virtual fluid balls from P to Q; Q moves more as compared to the P, so you have a stretching of fluid virtual fluid balls, stretching of the virtual fluid balls that means, they will stretch it, there will be the same dimensions but there will be stretching it, so they are will be elongated as the width will be reduces, the length will be change it.

So, you have this part, so increase in length of PQ like the segment yields a positive linear thing, when a flow is incompressible, the net volume of fluid element was to remain constant. If the fluid is incompressible that means, the density does not change within the fluid flow domain. It means that the net volume of the fluid must be constant now, if you look at these problems that so, you can find out this linear strain rate is very easily that linear strain rate length you know it divided by this per unit; increasing length you know it divided by this the original length will get it from that what will be the linear strain rate.

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So, we can compute it what would be that is why we can easily compute it, what it will be linear strain rate which is the gradient of the linear strain rate in the x directions, the y directions and the z direction. If I am looking for a poly metric strain rate that means, will be sum of these strain rates what we are looking at. Let me have a very simple examples how you have either; like you have this type of flow the conduits are you have.

When a fluid particle is coming like this or virtual ball is coming it, it has the velocity variation, as soon as its comes here, it has to be bigger size, so it have a is to occupy this space if you would have this, so these fluid particles from this point to A to B when passing through, it has gone through shear strain deformations. So, if you look at this fluid flow problems, which is very easy, things that there is a smaller diameter pipe, it is a connected to a bigger diameter pipe.

When a fluid elements is coming from this point as goes to this point has to be increased, there will be a linear strain rate what is going to happen in this case. So, because of that there will be reductions of fluid, so if we look at this way whenever the fluid particles passing through a smaller to bigger dimension or bigger to smaller dimension, it goes through a strain rate formations.

Volumetric strain rate in Cartesian coordinates:

$$\frac{1}{v}\frac{Dv}{Dt} = \frac{1}{v}\frac{dv}{dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Now, if you look at; if I look at a volumetric strain rate is a sum of the shear strain rate in the x, y and z directions, as you know it for incompressible flow, the volumetric strain rates becomes 0 that means, what is this; is a dot product of delta and v is equal to the 0, so this becomes a conservations equations; the mass conservation equation for us, if you consider the flow is incompressive.

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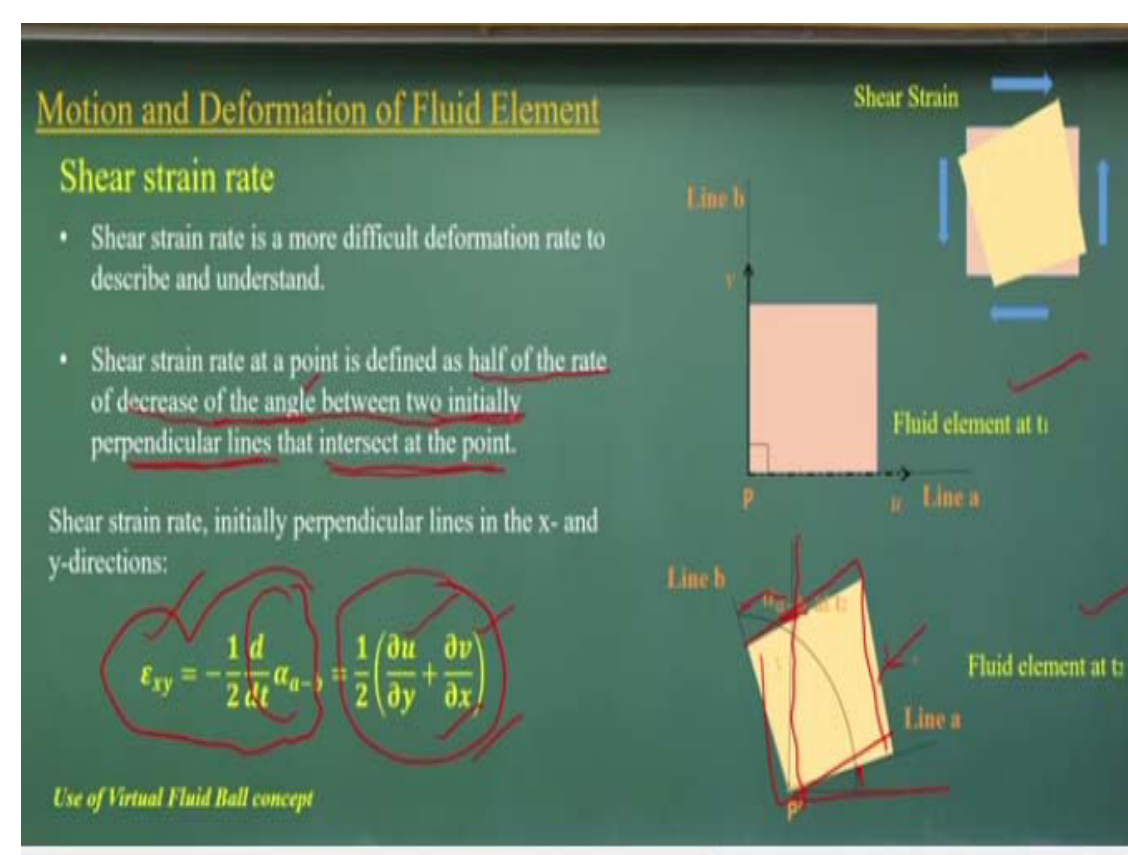
Now, let us go for the shear strain rate, what is this? The shear strain rate is a half rate of decrease of angle between 2 initially perpendicular lines that intersect at the point, what it is saying that if you look at these figures that I have a figure like this, the fluid element at the t1 time at the fluid element at the t2 time. At the t2 time, this line is moving α and α_{a-b} angles.

So, we are looking at half of that angles, the decrease of angle between 2 that means, because of shear strain rate, you can identify the age and how this age are changing it okay, on average how much of age it is a changing it with a 2 perpendicular lines, okay, you can understand it if I have the shear stress is acting it, the strain will be the angular rotations.

$$\varepsilon_{xy} = -\frac{1}{2}\frac{d}{dt}\alpha_{a-b} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

And since is the fluid element we will have to have a 2 axis and with that 2 axis formations we are looking at that what is the angle rate of the change of decrease of angle between 2 initially perpendicular line that what intersected that, so again you can find out what will be the shear strain rate with the half of this change in the α_{a-b}, okay that angle between these 2 change that is what again you can estimate it.

And you will find out the linear shear strain rate will be the functions of u and v, the scalar velocity components, the partial gradient component what you will get this function like this. So, you can easily understand it that if I have the fluid part; balls also can find out how much of rotations are happening it and because of if in case of angular rotations, both the axis are rotating in the same rate.

But in this case, the rotation rate are the difference because of that you can see that deformations, the fluid element deformations.

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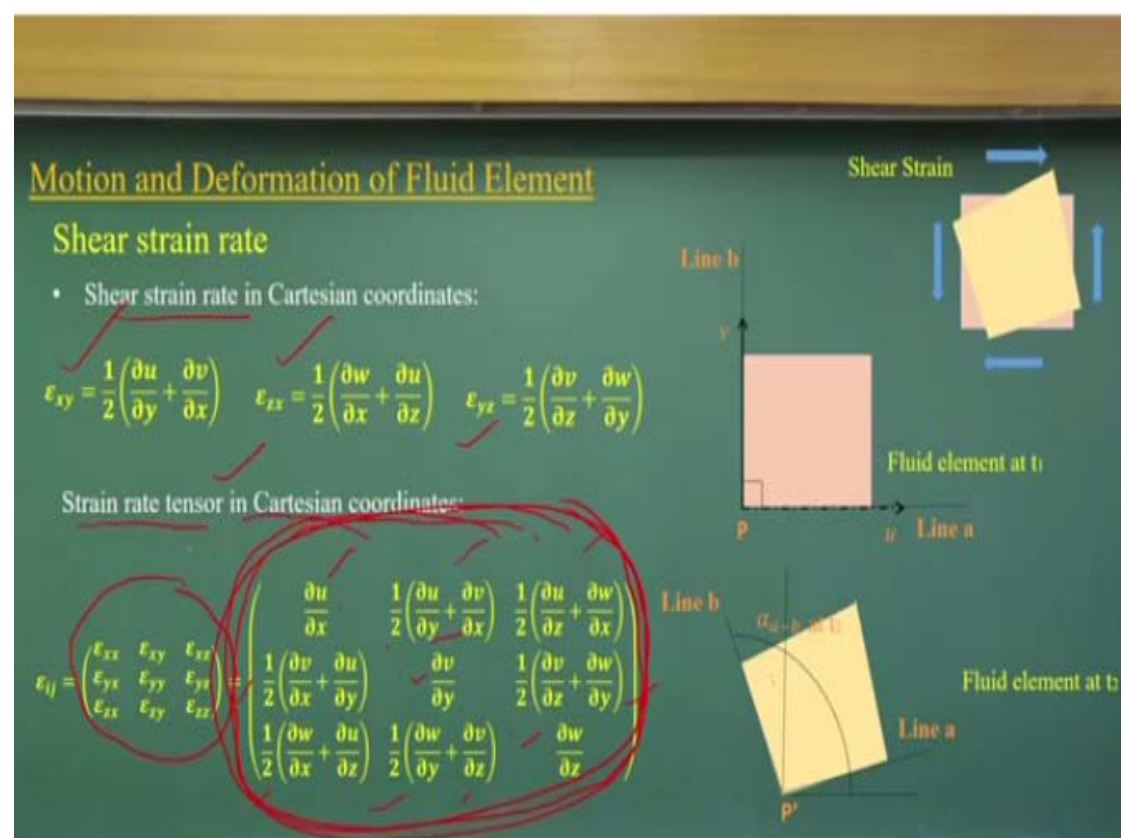
So, as of now what we have done it, we can get a shear strain rate in a Cartesian coordinate for different plane, xy plane, zx plane and yx plane and finally, we can write the shear strain rate tensor which will be linear strain formations and the formations of shear strain rate. So, it has a 9 component of stress strain rate tensors okay, so you can find out the strain rate tensor functions which would have a 9 components.

Shear strain rate in Cartesian coordinates:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

Strain rate tensor in Cartesian coordinates:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & \dfrac{\partial v}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) & \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) & \dfrac{\partial w}{\partial z} \end{pmatrix}$$

So, if you look at that this strain rate tensor is a functions of partial derivative of the u, v, w scalar velocity components, there could be positive, there could be linear part but they are all depends upon the velocity variations in x and yz of planes, so how would they velocity variation and that velocity variation causing as the shear strain tensors. So, as you know from the fluid mechanics or the solid mechanics, stress strain; stress is a proportional to the shear strain rate.

So, we can easily establish the stress and the shear strain rate tensor functions very easily, if you know the shear strain rate tensors but in these 20 hours' lectures, I am not going more details about these functions because it is limited to this way to just describe you how the shear strain rate functions are there.

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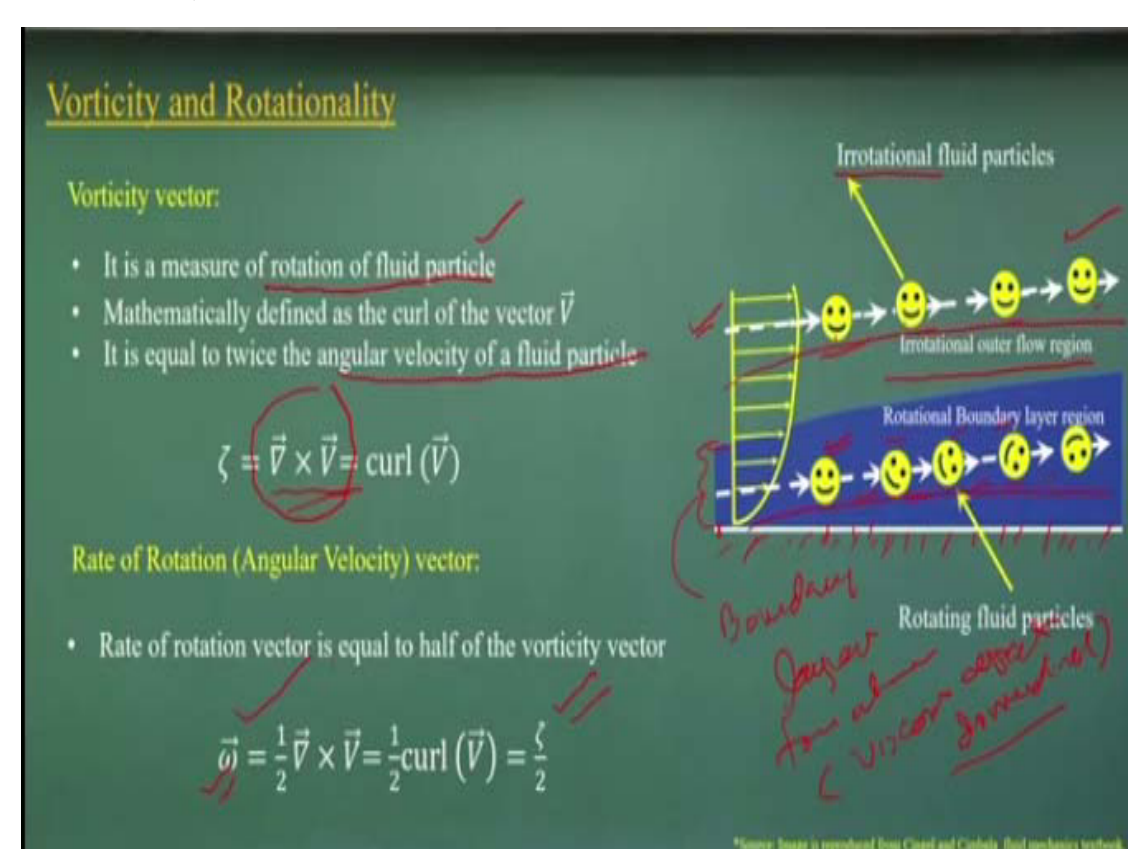
So, let us look at another component is called the vorticity or the rotationality, when you talk about a vorticity vectors, the basically we are representing as a vorticity vectors which actually the measures of rotations of a fluid particle, which if you can look at the rotations of fluid particles can be twice of the angular velocity, so it is the curl of V okay;

$$\zeta = \vec{\nabla} \times \vec{V} = \text{curl } (\vec{V})$$

and half of the curl of the V is as you know it is angular rotations, you just substitute the v functions and do the cross products, you will get the; divide by 2, you will get the rate of rotations.

$$\vec{\omega} = \frac{1}{2} \vec{\nabla} \times \vec{V} = \frac{1}{2} \text{curl } (\vec{V}) = \frac{\zeta}{2}$$

Or indirectly that it has the a relationship with a vorticity with the rotations because this is what again a representations, vorticity vector, if I know the cross product of the $\vec{\nabla}$ and V, which is 0, then we can say that the vorticity is 0 that means, there is no rotations, there is no angular rotations like for example, if you look at very interesting photographs or figures okay which is modified from Cengels and Cimbala's books.